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THE PROPAGATION OF LIGHT IN A LOW-ABSORBING DIELECTRIC LAYER

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The reflection and transmission of light waves by low- absorbing dielectric layer on semi-infinite dielectric substrate is considered. Shown that in this system under certain conditions may be realised the effect of antireflection of light. Expressions for selective layer thickness and frequencies in which occur reflectionless absorption of light have been obtained.

1. Introduction

Thin films optics deals with the propagation of light waves through single films and multilayers. The film is characterised by its refraction index, its absoption coefficient and its thickness.

Propagation of light inside the material is strongly affected by the periodicity. Study of light propagation in layered structures becomes actual recently also in connection with analysis of operation of lasers with distributed feedback and nanostructures. By playing with light interferences, it is possible to fabricate a Bragg mirror, using stacks of dielectric layers.

The theory of optical design of multilayers is quite complex and tedious, and has been presented for some simple cases [1-3]. For microwave region of spectrum the effect of reflectionless absorption in layered medium both theoretically and experimentally was investigated in [4,5]. This method of investigation was extended to include optical wavelengths in [6].

In this paper we consider reflection and transmission of optical waves by low-absorbing dielectric layer on semi-infinite non-absorbing dielectric substrate.

2.Basic equations

We start by considering the reflection and transmission of light in a absorbing dielectric layer on semi-infinite non-absorbing dielectric substrate. Incident light is considered normally and plane polarised. In order to find an expressions for the reflectance and transmittance of a dielectric film illuminated by a parallel beam of light at wavelength λ, we must consider the multiple reflections of light at each surface of the film and perform a multiple beam summation. Thus reflected and transmitted complex amplitudes are given by [2]

$$\hat{R} = \frac{\hat{r}_1 + \hat{r}_2 e^{-2ikl}}{1 + \hat{r}_1 \hat{r}_2 e^{-2ikl}} \ , \quad \hat{T} = \frac{\hat{t}_1 \hat{t}_2 e^{-ikl}}{1 + \hat{r}_1 \hat{r}_2 e^{-2ikl}} \tag{1}$$

where $\hat{r}_1 = r_1 e^{i\varphi_1}$; $\hat{r}_2 = r_2 e^{i\varphi_2}$; $r_1, r_2 . \varphi_1, \varphi_2$ are complex amplitudes, modules and phases of reflection coefficients, respectively, k is complex wave number. In these equations the subscripts 1and 2 refer to the first and second surface of the layer. The layer is assumed to be plane and parallel sided of thickness l and complex refractive index $\hat{n}$ and is bounded by semi-infinite layers of indices 1 and n_1.

The Fresnel coefficients of reflection and transmission are

$$\hat{r}_1 = \frac{1-\hat{n}}{1+\hat{n}}, \quad \hat{r}_2 = \frac{\hat{n}-n_1}{\hat{n}+n_1}, \ \hat{t}_1 = \frac{2}{1+\hat{n}}, \ \hat{t}_2 = \frac{2\hat{n}}{n_1+\hat{n}} \tag{2}$$

In formula (1), propagation constant k of the wave traveling in the covering material is

$$k = \frac{2\pi}{\lambda}(n - i\chi) = \frac{2\pi}{\lambda_d}(1 - iy) \tag{3}$$

where $\lambda_d = \lambda / n$, $y = \chi / n$, n is the refractive index of the layer material, χ is the extinction coefficient of the material. The relation between extinction χ and absorption $\alpha(\lambda)$ coefficients in given wavelength λ is

$$\alpha(\lambda) = \frac{4\pi\chi}{\lambda} \tag{4}$$

For the concidered layered structures modules and phases of reflection coefficients are given by the well-known relations

$$r_1 = \sqrt{\frac{(1-n)^2 + \chi^2}{(1+n)^2 + \chi^2}} \ , \qquad \varphi_1 = arctg \frac{2\chi}{1 - n^2 - \chi^2}$$

$$r_1 = \sqrt{\frac{(1-n)^2 + \chi^2}{(1+n)^2 + \chi^2}} \ , \qquad \varphi_2 = arctg \frac{2n_1\chi}{n_1^2 - n^2 - \chi^2} \tag{5}$$

Let introduce $x = l / \lambda_d$. The expressions for energy reflectance and transmittance may be obtained from equation (1)

$$R^2 = \frac{\left(r_1 - r_2 e^{-4\pi xy}\right) + 4 r_1 r_2 e^{-4\pi xy} \cos^2\left(2\pi x + \frac{\varphi_1 - \varphi_2}{2}\right)}{\left(1 - r_1 r_2 e^{-4\pi xy}\right)^2 + 4 r_1 r_2 e^{-4\pi xy} \cos^2\left(2\pi x + \frac{\varphi_1 + \varphi_2}{2}\right)} \qquad (6)$$

$$T^2 = \frac{(1 - r_1)(1 - r_2) e^{-4\pi xy}}{\left(1 - r_1 r_2 e^{-4\pi xy}\right)^2 + 4 r_1 r_2 e^{-4\pi xy} \cos^2\left(2\pi x + \frac{\varphi_1 + \varphi_2}{2}\right)} \qquad (7)$$

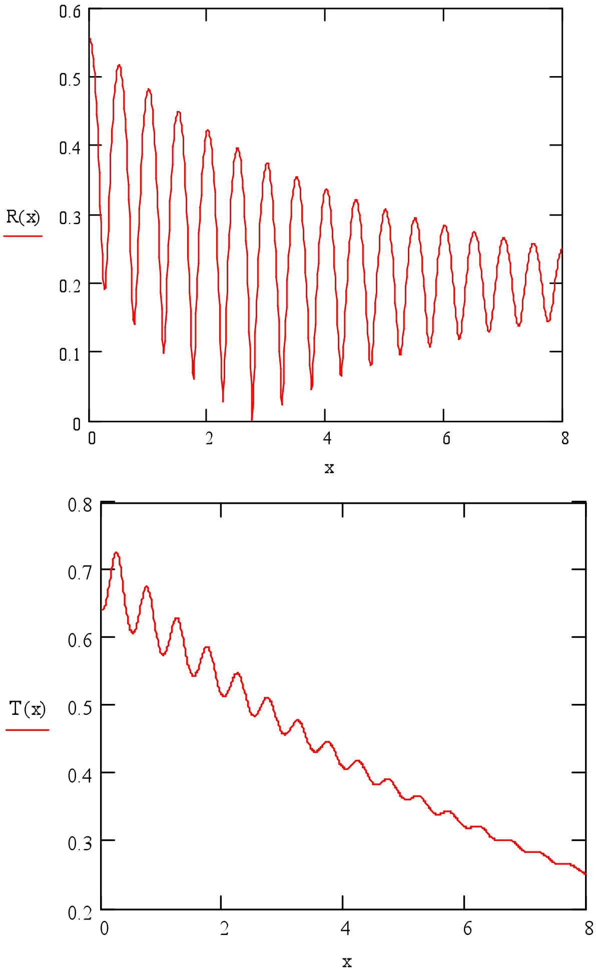

Fig.1 Absolute values of reflection R(x) and transmission T (x) coefficients .vs. thickness of the covering material in the two-layer dielectric-dielectric structure: $r_1 = 0.2$; $r_2 = 0.4$; $n = 1.5$; $y = 0.02$.

Fig.1 represents dependence of reflection coefficient amplitude R and transmission coefficient amplitude T on absorbing layer thickness when the coating is low-absorbing. One can see that reflection coefficient has damping oscillations. Thus a standing of extremes of curves $R(l)$ are realized at thickness of a absorbing layer of distinct from quantities of multiple $\lambda_d/4$ .There are two regions in the $R(l)$ dependence which differ by character of changing of extrema in increasing of thickness of the layer. The lower absorption coefficient the higher the number of minimum which takes place no reflection.

From equation (1) one can see that the condition of reflectionless absorption of incident light is

$$r_2\exp(i\varphi_2-2ikl)=-r_1\exp(i\varphi_1) \tag{8}$$

It is well – known that in its simplest form (when $\chi=0$) anti- reflecting layer is homogeneous, its thickness is a quarter wavelength and its refractive index equals the geometric mean of the indices of the two adjoining media :

$$n=\sqrt{n_1} \tag{9}$$

In this connection we shall introduce the suggestion that the specified zero minimum of function$R(l)$ is realized at thickness of a layer of the substance a little differing from quantities of multiple $\lambda_d/4$

$$x=\frac{2N-1}{4}+\Delta \tag{10}$$

where N is ordinal number of the zeroth minimum of function $R(l)$; Δ is the quantity to be determined by optical parameters of coating material.

Substituting equations (3) and (10) into equation (8), we find

$$\ln\frac{r_2}{r_1}=y[\pi(2N-1)+\varphi_2-\varphi_1] \tag{11}$$

$$\frac{l}{\lambda}=\frac{1}{n}\left(\frac{2N-1}{4}+\frac{\varphi_2-\varphi_1}{4\pi}\right) \tag{12}$$

$$\Delta=\frac{\varphi_2-\varphi_1}{4\pi} \tag{13}$$

Equations (10)- (12) allow functional relation between incident light wavelength λ, layer thicknees l and its optical parameters n and χ, for to arise reflectionless absorption in the coating.

It is clear that to calculate the selective quantities of wavelength λ_0 and thickness l_0 which reflectionless absorption occurs we have to take into account spectral characteristics of the coating.

3.Dispersion relations for dielectric layer

According to dispersion theory of light the real ε' and imaginary parts ε'' of dielectric constant may be expressed as [2]

$$\varepsilon' = n_\infty^2 + \frac{4\pi N_0 q^2}{m}\frac{\omega_1^2-\omega^2}{\left(\omega_1^2-\omega^2\right)^2+\gamma^2\omega^2}; \quad \varepsilon'' = \frac{4\pi N_0 q^2}{m}\frac{\gamma\omega}{\left(\omega_1^2-\omega^2\right)^2+\gamma^2\omega^2} \qquad (14)$$

where n_∞ is refractive index far from the resonance; q, m are charge and mass of electron, N_0 is concentration, γ is damping coefficient, ω is frequency. Resonance frequency ω_1 according to Lorentzs field in condenced matter may be expressed by frequency ω_0 and concentration

$$\omega_1^2 = \omega_0^2 - \frac{4\pi N_0 q^2}{m} \qquad (15)$$

It is easy in the first approximation from (14) to have relation between ε' and ε''

$$\left(\varepsilon' - n_\infty^2\right)^2 + \left(\varepsilon'' - b\right)^2 = b^2 \qquad (16)$$

where

$$b = \frac{2\pi N_0 q^2}{m\gamma\omega_1} \qquad (17)$$

Solution of equations (11) and (16) allows us to find optical parameters of absorbing dielectric layer

$$\varepsilon' = n^2 - \chi^2 = n^2\left(1 - y^2\right), \ \varepsilon'' = 2n\chi = 2n^2 y \qquad (18)$$

To determine the frequency which the effect of reflectionless absorption takes place may be used next equation

$$\frac{\varepsilon''}{\varepsilon' - n_\infty^2} = \frac{\gamma}{2\left(\omega_1 - \omega\right)} \qquad (19)$$

For to find thickness of layer may be used the equation (12).

We have been presented basic expressions to solve the problem of reflection and transmission in absorbing coating. But in learn this problem by practice often we deal with low – absorbing layers $(\chi << n)$ When $(\chi << n)$ from equations (5) and (18) may be obtained

$$r_1 = \frac{n-1}{n+1}, \quad r_2 = \frac{n-n_1}{n+n_1}; \varphi_2 - \varphi_1 = \frac{2n_1\chi}{n_1^2 - n^2 - \chi^2} - \frac{2\chi}{1-n^2-\chi^2}; \varepsilon' = n^2 \quad 20)$$

From equations (11) and (20) in the first approximation one can see the next relation

$$\chi = \frac{1}{\pi(2N-1)} \frac{2n\left(n_1 - n^2\right)}{(n-1)(n+n_1)} \tag{21}$$

Combined solution of equations (16) and (21) allows to determine selective quantities of n and χ for low – absorbing dielectric layer. Then from equations (12) and (19) we can find frequency and thickness of layer to be obtain reflectionless absorption.

When $\chi = 0$, $\Delta = 0$ we have from equations (9) and (10)

$$n_1 = n^2, \; l_0 = \frac{\lambda_0}{4n}(2N-1) \tag{22}$$

which are well – known conditions in transparency optics [2].

The energy transmitted from low – absorbing layer in the reflectionless absorption may be expressed by

$$\tau = n_1\left|\hat{T}\right|^2 = n_1\sqrt{\frac{\left(1-n^2\right)^2 + 2\chi^2\left(1+n^2\right)}{\left(n_1^2 - n^2\right)^2 + 2\chi^2\left(n_1^2 + n^2\right)}} \tag{23}$$

Optical parameters n_1, n and χ in (23) are quantities which satisfied condition of reflectionless absorption and related in equation (11).

4.Conclusion

The reflection and transmission of light by absorbing two-layer dielectric- dielectric structure is theoretically considered. Expressions for selective layer thickness and frequencies in which occur reflectionless absorption of light in this system have been obtained. The conditions for to realize anti-reflecting coatings in optical medium with resonance type dispersion have been found.

Propagation of light in low-absorbing layered system have been analysed. The energy transmitted from low-absorbing layer when reflectionless absorption takes place was calculated. The lower absorption coefficient of layer the higher the number of minimum for no reflection of light in the dependence of reflection coefficient on layer thickness.

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ZƏİF UDAN DİELEKTRİK TƏBƏQƏDƏ İŞIĞIN YAYILMASI

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XÜLASƏ

Sonsuz qalınlığa malik dielektrik üzərinə çəkilmiş udan dielektrik təbəqədən işığın əks olunması və keçməsinə baxılmışdır. Göstərilmişdir ki, müəyyən şərtlər daxilində bu sistemdə işığın əks olunmadan udulması mümkündür. İşığın əks olunmadan udulmasına uyğun lövhə qalınlığı və tezliklər üçün ifadələr alınmışdır.

РАСПРОСТРАНЕНИЕ СВЕТА В СЛАБОПОГЛОЩАЮЩЕМ ДИЭЛЕКТИЧЕСКОМ СЛОЕ

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РЕЗЮМЕ

Рассмотрено отражение и пропускание световых волн слабопоглощающим диэлектрическом слоем на полубесконечной диэлектрической подложке. Показано, что при определенных условиях в этой системе возможно безотражательное поглощение света. Получены выражения для селективных значений толщины слоя и частоты света при которых происходит безотражательное поглощение.